Mechanism of hypersensitive transport in tilted sharp ratchets

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The noise-flatness-induced hypersensitive transport of overdamped Brownian particles in a tilted ratchet system driven by multiplicative nonequilibrium three-level Markovian noise and additive white noise is considered. At low temperatures, the enhancement of current is very sensitive to the applied small static tilting force. It is established that the enhancement of mobility depends nonmonotonically on the parameters (flatness, correlation time) of multiplicative noise. The optimal values of noise parameters maximizing the mobility are found.

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Recently, noise-induced hypersensitivity to small time-dependent signals in nonlinear systems with multiplicative noise has been the topic of a number of physical investigations [1–4]. A motivation in this field has come from numerical, analytical, and experimental studies of a nonlinear Kramers oscillator with multiplicative white noise [1,4]. Under the effect of intense multiplicative noise, the system is able to amplify an ultrasmall deterministic ac signal (of the order of, e.g., 10^{-20}) up to the value of the order of unity [1]. Afterwards, a related phenomenon such as noise-induced hypersensitive transport was found in some other systems with multiplicative dichotomous noise [2,3]. Noise-induced hypersensitive transport was also established in a phase model, i.e., $d\varphi/dt = a - b\sin\varphi$, with a strong symmetric multiplicative colored noise. It was shown that in such a system, a macroscopic flux (current) of matter appears under the effect of ultrasmall dc driving [2]. It is important to notice that the physical mechanism underlying the phenomenon of hypersensitive transport presented in Refs. [2,3] is based on the assumption that the periodic potential is smooth. It is easy to see that in the case of a periodic sharp potential, the above mechanism cannot bring forth any hypersensitive transport.

Theoretical investigations [5–9] indicate that noise-induced nonequilibrium effects are sensitive to noise flatness, which is defined as the ratio of the fourth moment to the square of the second moment of the noise process. Although its significance is obvious, the role of the flatness of fluctuations has not been researched to any significant degree to date. In the present paper, we assume the multiplicative noise to be a zero-mean trichotomous Markovian stochastic process [10]. It is remarkable that for trichotomous noises, the flatness parameter φ, contrary to the cases of the Gaussian colored noise ($\varphi = 3$) and symmetric dichotomous noise ($\varphi = 1$), can have any value from 1 to ∞. The flatness as an extra degree of freedom (in comparison with dichotomous noise) can prove useful when modeling actual fluctuations, e.g., thermal transitions between three configurations or states. This is the reason why we choose in the phase space of possible nonequilibrium models the trichotomous noise. Although both dichotomous and trichotomous processes may be too rough approximations of the actual nonequilibrium fluctuations, the latter is more flexible, including all cases of dichotomous processes and, as such, revealing the essence of its peculiarities. A further virtue of the models with trichotomous noise is that they constitute a case admitting exact analytical solutions for some nonlinear stochastic problems, such as colored-noise-induced transitions [10] and reversals of noise-induced flow [9].

The main purpose of this paper is to establish a mechanism of hypersensitive transport, demonstrating that the flatness of multiplicative noise can generate hypersensitive response to the small external static force in a tilted sharp ratchet system. We will show that in the region of hypersensitive response, the value of mobility can be controlled by means of thermal noise. For low temperatures, we find that the mobility exhibits resonant behavior at intermediate values of the parameters of the multiplicative noise (flatness, correlation time).

We consider an overdamped multinoise tilted ratchet, where particles move in a one-dimensional spatially periodic potential of the form $V(x,t) = V(x)Z(t)$, where $Z(t)$ is a trichotomous process [10] and $V(x)$ is a piecewise linear function, which has one maximum per period. The additional force consists of thermal noise with temperature D, and an external static force F. The system is described by the dimensionless Langevin equation

$$\frac{dX}{dt} = Z(t)h(X) + F + \xi(t), \quad h(x) \equiv -\frac{dV(x)}{dx}, \tag{1}$$

where $V(x) = \tilde{V}(\tilde{x})/\tilde{V}_0, \tilde{V}(\tilde{x})$ is a spatially periodic function with period $\tilde{L}$, and $\tilde{V}_0 = \tilde{V}_{max} - \tilde{V}_{min}$. The usual dimensionalized physical variables are indicated by tildes and the space and time coordinates read $X = \tilde{X}/\tilde{L}$ and $t = \tilde{t}\tilde{V}_0/\kappa\tilde{L}^2$ with κ being the friction coefficient; $\tilde{F} = \tilde{V}_0 F/\tilde{L}$ is a constant external force. The thermal noise satisfies $\langle \xi(t) \rangle = 0$ and $\langle \xi(t_1)\xi(t_2) \rangle = 2D\delta(t_1 - t_2)$. Regarding the random function $Z(t)$, we assume it to be a zero-mean trichotomous Markovian stochastic process [10] which consists of jumps among three values $z = \{1, 0, -1\}$. The jumps follow in time according to a Poisson process, while the values occur with the stationary probabilities $P_s(1) = P_s(-1) = q$ and $P_s(0) = 1$

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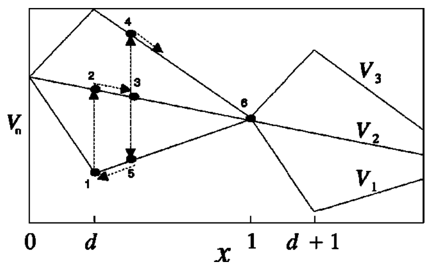

FIG. 1. The mechanism of hypersensitive transport. The lines depict the net potentials $V_n(x)=z_nV(x)-Fx$ with $z_1=1$, $z_2=0$, and $z_3=-1$. A particle cannot move of its own accord along the potentials V_1 and V_3. However, if one allows switching between the potentials V_n, $n=1,2,3$, the particle will move downhill along the trajectory 1:2:3:4:6.

$-2q$. In a stationary state, the fluctuation process statisfies $\langle Z(t)\rangle=0$ and $\langle Z(t+\tau)Z(t)\rangle=2q\exp(-\nu\tau)$, where the switching rate ν is the reciprocal of the noise correlation time $\tau_c=1/\nu$. The trichotomous process is a special case of the kangaroo process [5] with a flatness parameter $\varphi=\langle Z^4(t)\rangle/\langle Z^2(t)\rangle^2=1/(2q)$. At large flatnesses, our trichotomous noise essentially coincides with the three-level noise used by Bier [6] and Elston and Doering [7].

The master equation corresponding to Eq. (1) reads

$$\frac{\partial}{\partial t}P_n(x,t)=-\frac{\partial}{\partial x}[\Gamma_nP_n(x,t)]+\sum_m U_{nm}P_m(x,t), \quad (2)$$

where $\Gamma_n=z_n\,h(x)+F-D\,\partial_x$ and $P_n(x,t)$ is the probability density for the combined process (x,z_n,t), $n,m=1,2,3$, $z_1=1$, $z_2=0$, $z_3=-1$, and $U_{ik}=\nu[q+(1-3q)\delta_{i2}-\delta_{ik}]$. The stationary current $J=\Sigma_nj_n(x)$ is then evaluated via the current densities $j_n(x)=[z_nh(x)+F-D\,\partial_x]P_n^s(x)$, where $P_n^s(x)$ is the stationary probability density in the state (x,z_n). To calculate the stationary probability density in the x space, $P(x)=\Sigma_nP_n^s(x)$, and the stationary current, $J=$const, six conditions are imposed on the solutions of Eq. (2), namely, the conditions of periodicity $P_n^s(x)=P_n^s(x+1)$, $n=1,2,3$, and normalization of $P_n^s(x)$ over the period interval $L=1$ of the ratchet potential $Z(t)V(x)$, which read $\int_0^1P_1^s(x)dx=\int_0^1P_3^s(x)dx=q$ and $\int_0^1P_2^s(x)dx=1-2q$.

To derive an exact formula for J, we assume that the potential $Z(t)V(x)=Z(t)V(x-1)$ in Eq. (1) is piecewise linear (sawtoothlike) and its asymmetry is determined by a parameter $d\in(0,1)$, with $V(x)$ being symmetric when $d=1/2$. A schematic representation of the three configurations assumed by the "net potentials" $V_n(x)=z_nV(x)-Fx$ associated with the right hand side of Eq. (1) is shown in Fig. 1. Regarding the symmetry of the dynamic system (1), we notice that $J(-F)=-J(F)$ and $J(F,d)=J(F,1-d)$. Thus we may confine ourselves to the case $d\leqslant1/2$ and $F\geqslant0$. Obviously, for $F=0$, the system is effectively isotropic and no current can occur. In the case of zero temperature, both the noise levels $z_{n=1,3}=\pm1$ in Eq. (1), where $F\leqslant\min\{1/d,1/(1-d)\}$, give zero flux. However, if one allows switching between the three dynamic laws $V_n(x)$, $n=1,2,3$, the resulting motion will have a net flux which can be much greater that the flux by the dynamic law $V_2=-Fx$. If the rate of reaching the minimal energy in each well considerably exceeds the switching rate ν, the leading part of the net flux is achieved in the following way: a particle locked in the potential minimum 1 switches to point 2, then slowly moves to point 3, switches to point 4 (or to 5 with equal probability), and rapidly slides down to point 6 (or from 5 back to 1), etc. (see Fig. 1 and cf. Ref. [11]). In this case, hypersensitive transport is possible and can be intuitively understood. The described scheme is valid only in the absence of thermal noise. Otherwise, a particle is able to pass by a thermally activated escape across the potential barriers in both directions. However, it predominantly moves to the right and hypersensitive transport still occurs (at least at sufficiently low temperatures). As the "force" $h(x)=-dV(x)/dx$ is piecewisely constant, $h(x)=h_1=1/d$ for $x\in(0,d)(\text{mod}1)$ and $h(x)=h_2=-1/(1-d)$ for $x\in(d,1)$ (mod1), Eq. (2) splits up into two linear differential equations with constant coefficients for the two vector functions $\mathbf{P}_i^{\mathbf{s}}(x)=(P_{1i}^s,P_{2i}^s,P_{3i}^s)$ $(i=1,2)$ defined on the intervals $(0,d)$ and $(d,1)$, respectively. The solution reads

$$P_{ni}^s(x)=JA_n+\sum_{k=1}^{5}C_{ik}A_{nik}e^{\lambda_{ik}x}, \quad (3)$$

where C_{ik} are constants of integration, the constants A_n and A_{nik} are given by $A_1=A_3=qJ/F$, $A_2=(1-2q)J/F$, $A_{nik}=(D\lambda_{ik}-F)[D\lambda_{ik}^2-(F-z_nh_i)\lambda_{ik}-\nu]$ for $n=1,3$, $A_{2ik}=2h_i^2\lambda_{ik}-(A_{1ik}+A_{3ik})$, and $\{\lambda_{ik},k=1,\ldots,5\}$ is the set of roots of the algebraic equation

$$D^3\lambda_i^5-3D^2F\lambda_i^4+D(3F^2-2D\nu-h_i^2)\lambda_i^3+F(4D\nu-F^2+h_i^2)\lambda_i^2+\nu(D\nu-2F^2+2qh_i^2)\lambda_i-\nu^2F=0. \quad (4)$$

Eleven conditions for the ten constants of integration of Eq. (3) and for the probability current J can be determined at the points of discontinuity, by requiring continuity, periodicity, and normalization of $\mathbf{P}_i^{\mathbf{s}}(x)$. This procedure leads to an inhomogeneous set of 11 linear algebraic equations. Now, an exact formula for the current J can be obtained as a quotient of two determinants of the 11th degree. The exact formula, being complex and cumbersome, will not be presented here, however, it will be used to find (i) the dependence of the current J on the tilting force F and the dependence of the mobility $m=J/F$ on the flatness $\varphi=1/(2q)$, which are displayed in Figs. 2 and 3, respectively, and (ii) the asymptotic limits of the current J at low temperature and small external force.

Figure 2 shows the induced current J as a function of the external force F for two different values of temperature and for three different values of d with fixed $\varphi=2.5$ and $\nu=8$. In this figure, one also observes the hypersensitive response at very low forcing, which apparently gets more and more pronounced as the thermal noise strength D decreases. For the case $D=0,d=0.5$, the results of Monte Carlo simulations of the current $J=J(F)$ are also presented. The tendency apparent in Fig. 2, namely, a decrease in the mobility for very low

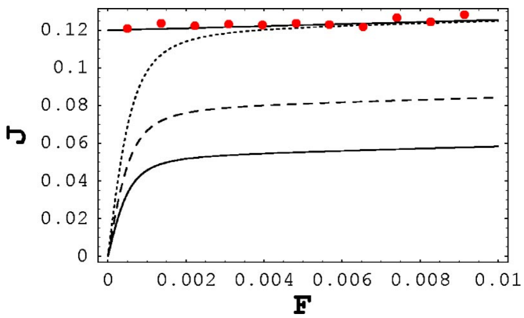

FIG. 2. The current J vs applied force F in the region of the hypersensitive response. The flatness parameter equals $\varphi=2.5$ and the switching rate $\nu=8$. Solid straight line: $D=0$, $d=0.5$. Dotted line: $D=4\times10^{-8}$, $d=0.5$. Dashed line: $D=4\times10^{-8}$, $d=0.2$. Solid curved line: $D=4\times10^{-8}$, $d=0.05$. The filled dots on the solid straight line are obtained by means of Monte Carlo simulations. Notice the jump of the current from the zero level to the solid line corresponding to the infinite derivative of $J(F)$ at $F=0$.

forcing as the asymmetry of the potential grows, is also valid for large asymmetries, e.g., when $d<0.05$.

To obtain more insight, we shall now study some asymptotic limits of the current.

At the fast-noise limit, we allow ν to become large, holding all the other parameters fixed. Thus, at very high frequencies of colored fluctuations, the system is under the influence of the average fluctuating potential. In the $\nu\to\infty$ limit, the current is then given by

$$J=F+O(\nu^{-1/2}).$$

The form of the leading term of the current J is not confined to the fast-noise limit. It is also valid for the asymptotic limit of a high temperature, $D\gg1$, and in the case of a large "load" force F ($F\to\infty$, all the other parameters fixed).

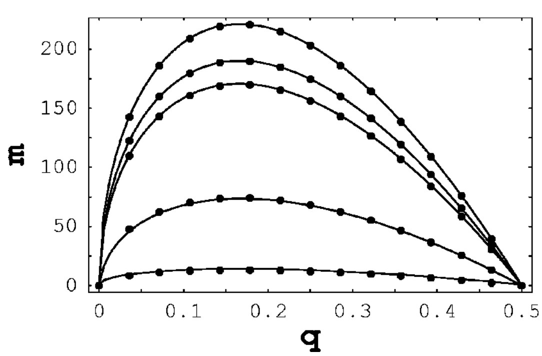

FIG. 3. The mobility $m=J/F$ vs the flatness parameter $q=1/(2\varphi)$ at $d=1/2$, $D=4\times10^{-8}$, and $F=10^{-5}$. The curves computed from the exact formula for the current J correspond to the values of $\nu=8/3$, $\nu=1$, $\nu=8$, $\nu=0.1$, $\nu=100$ from top to bottom. The nonmonotonic sequence of the values of ν stems from the bell-shaped dependence $J=J(\nu)$. Note that the maximum of the mobility lies at $q=1/6$ and $\nu=8/3$. The dots were computed by means of the asymptotic formula (7).

At the long-correlation-time limit $\nu\to0$, Eqs. (2) for $P_1^s(x), P_2^s(x)$, and $P_3^s(x)$ are decoupled and the total current is given by the average of each current for the corresponding potential configurations. In the case of the symmetric potential $d=1/2$, the current J saturates at the value

$$J=(1-2q)F+\frac{2q(4-F^2)^2\sinh(F/2D)}{16D\left[\cosh\dfrac{1}{D}-\cosh\dfrac{F}{2D}\right]-F(4-F^2)\sinh\dfrac{F}{2D}}.$$

For $F<2$, we can see that the current J tends to $(1-2q)F$ as $D\to0$. This result is consistent with the physical intuition that the probability densities $P_1^s(x)$ and $P_3^s(x)$ are δ distributed at deterministic stationary states (minima of potentials): the random variable Z takes values ±1 for a sufficiently long time to allow the deterministic stationary state to be formed.

In the case of zero temperature $D=0$ and symmetric potential $d=1/2$, one finds from the exact formula that on the assumption $F<2$ the current equals

$$J=\nu F\frac{A_1C_2-C_1A_2}{B_1C_2-B_2C_1}, \tag{5}$$

where $A_i=F(\alpha_i[F-(4-F^2)\eta_i]-2(1-2q))$, $B_i=(\nu+16q)A_i+32q(1-2q)(2\alpha_i+F)$, $C_i=qA_i+2(1-2q)[4\eta_i+Fq+2\alpha_i(1+F\eta_i)]$, $\eta_i=F^{-1}(4-F^2)^{-1}(F^2-4q-2\varepsilon_i[4q^2+F^2(1-2q)]^{1/2})$, $\alpha_i=\tanh(\nu\eta_i/4)$, $i=1,2$, $\varepsilon_1=1$, $\varepsilon_2=-1$.

Thus, at the low-force limit $F\to0$, the current will saturate at the finite value

$$\lim_{F\to0}J=J_a=\frac{32\nu q(1-2q)}{(\nu+8)^2}. \tag{6}$$

As $J(F=0)=0$, the hypersensitive response is extremely pronounced in this case, with the current picking up with an infinite derivative at $F=0$ (see also Fig. 2). The asymptotic current J_a exhibits a bell-shaped (resonance) form as ν or q is varied. The optimal correlation time τ_m that maximizes the current equals 1/8, and the optimal flatness $\varphi_m=1/(2q_m)=2$. It is remarkable that in the case of a dichotomous noise $q=1/2$, the hypersensitive response disappears and in the low-forcing limit the leading-order term of the current is proportional to F: $J\approx\nu F(\nu+12)/(\nu+8)^2$.

At the low-forcing limit $F\ll1$, a natural way to investigate the behavior of J is to apply small-F perturbation expansions. A stationary solution of Eqs. (3) and (4) with $D\neq0$, $d=1/2$ is constructed in terms of integer powers of F. The current can be expressed as $J=Fm_1+F^2m_2+\cdots$. We shall calculate the leading term of the current Fm_1. Notably, the analysis of this section is valid for the values of parameters satisfying the condition $F<(2q\nu D)^{1/2}$. This condition results from the assumption that the higher-order terms in the expansion of the roots of Eq. (4) are asymptotically smaller than the lower-order terms held in the calculation. At sufficiently small temperature $D\ll\min\{1,2q\nu,8q/\nu\}$, the formula for the leading-order term Fm_1 of the current is

$$J \approx Fm_1 = \frac{8(1-2q)F}{(\nu+8)^2}\sqrt{\frac{2q\nu}{D}} + FG. \tag{7}$$

Here the symbol G stands for the terms which do not increase as $D \to 0$. An extreme sensitivity of the mobility m to thermal noise can be seen from the factor $D^{-1/2}$ in Eq. (7) that increases unboundedly as $D \to 0$. It can be seen easily that the functional dependence of the mobility on the flatness φ and on the correlation time τ_c is of a bell-shaped form. The mobility m_1 reaches a maximum at the flatness $\varphi_m = 3$ and at the correlation time $\tau_m = 3/8$. The dependence of the mobility $m = J/F$ on the parameters q and ν for a fixed force value $F = 10^{-5}$ and for a fixed temperature $D = 4 \times 10^{-8}$ is shown in Fig. 3. We can see that the asymptotic formula (7) is in excellent agreement with the exact results.

Let us note that the sufficient condition $F < \sqrt{2q\nu D}$ has a distinct physical meaning: the characteristic distance of thermal diffusion $\sqrt{D/\nu}$ is larger than the typical distance F/ν for the particle driven by the deterministic force F in the state $z = 0$ of the trichotomous noise. Let us look at the latter statement more closely on the assumption that $\nu \ll 1$. For this assumption within the interval (0,1), the probability distributions $P_n^s(x)$, $n = 1,3$, are, evidently, concentrated at $x = 0$ (or $x = 1/2$). Next, we shall consider the trajectory $(1{:}2{:}3{:}4{:}6)$ in Fig. 1. The particles locked at the potential minimum 1 ($x = d = 1/2$) will go at the initial time $t = 0$ to point 2, where $z = 0$. The first time when the noise turns to either $z = 1$ or $z = -1$ is denoted by t_0. As the time of movement from 4 to 6 is much less than t_0, it is easy to find that during the time interval $(0,t_0)$ the center of mass has shifted by

$$\Delta x \approx \frac{1}{2\sqrt{\pi D t_0}} \int_0^{Ft_0} \exp\left\{-\frac{(x - Ft_0)^2}{4Dt_0}\right\} dx \approx \frac{F\sqrt{t_0}}{2\sqrt{D\pi}}.$$

In the case of a trichotomous noise, the probability $W(t)$ that in a certain time interval $(0,t)$ the transitions $z = 0 \to z = \pm 1$ do not occur is given by $W(t) = \exp(-2q\nu t)$. The probability that the transition $z = 0 \to z = -1$ occurs within the time interval $(t, t+dt)$ is $q\nu dt$. Consequently,

$$\langle \Delta x \rangle = q\nu \int_0^\infty e^{-2q\nu t_0} \Delta x \, dt_0 \approx \frac{F}{8\sqrt{2q\nu D}}.$$

Considering that the average number of transitions per unit of time into the state $z = 0$ is $2q\nu(1-2q)$, we obtain $J = 2q\nu(1-2q)\langle \Delta x \rangle \approx F(1-2q)\sqrt{2q\nu}/8\sqrt{D}$. Thus, we have obtained an earlier result, namely, Eq. (7) for $\nu \ll 1$. Formula (7) is one of our main results. Note that the above procedure can be repeated in a straightforward but tedious way for more complicated cases involving asymmetric potentials and potentials with several extrema per period. The phenomenon is robust enough to survive a modification of the multiplicative noise. The key-factor is the noise flatness, indicating how long the noise level dwells on the state $z = 0$. If the flatness parameter is greater than 1, the effect does exist. For example, the multiplicative noise can also be a Gaussian stationary process.

It is quite remarkable that the above results are also applicable for amplifying adiabatic time-dependent signals, i.e., signals of much greater periods than the characteristic time of establishing a stationary distribution, even in the case of a small input signal-to-noise ratio $F/\sqrt{D} \ll 1$.

We emphasize that our mechanism of hypersensitive transport is of a qualitatively different nature from a recently found effect, where a noise-induced enhancement of the current of Brownian particles in a tilted ratchet system has also been established [2,3]. In the latter scenario, a system with a periodic smooth potential exhibits hypersensitivity under the effect of multiplicative dichotomous noise because of noise-induced escape through fixed points of the dynamics. This occurs because the stable and unstable fixed points of the alternative dynamics, which coincide in the absence of the tilt F, are shifted apart by a small force (see also Ref. [11]). In the mechanism reported here, we have a sharp periodic potential: the stable and unstable fixed points of the dynamics coincide also for any small tilt. The crossing of the location of the fixed points is achieved by a combined influence of the flatness of the multiplicative noise and a small tilt forcing.

In a general case, if the potential is smooth and the flatness of multiplicative noise is greater than 1, both mechanisms play an important role and should be taken into account. Our calculations show that the factor $F\sqrt{\nu/D}$ in Eq. (7) is generated by thermal diffusion in the state $z = 0$, while the circumstance that the potential is sharp has no effect on this factor. On the other hand, for adiabatic switching, the mechanism described in Ref. [2] generates the current $J \sim \nu F/\sqrt{D}$. Consequently, our mechanism for sufficiently small switching rates induces hypersensitive transport more effectively than the one proposed by Ginzburg and Pustovoit. This conclusion is in agreement with the results of Ref. [2], presenting numerical simulations of the phenomenon of hypersensitive transport based on a phase model with the multiplicative colored Gaussian noise ($\varphi = 3$). It is established that in the case of low switching rates, the transport for the Gaussian noise appears to be more effective than for dichotomous stimuli. Regrettably, the authors of Ref. [2] did not consider the role of noise flatness and the physics of this discrepancy.

In conclusion, the reported mechanism of generating hypersensitive transport by the flatness of multiplicative noise is of general relevance for many physical, biological, and chemical systems, and may provide another possibility to control signal amplification. The sensitivity of system response to small input signals can be either enhanced or suppressed by changing the noise parameters (flatness, correlation time, temperature). In agreement with Ref. [12], we believe that the phenomenon proposed may also shed some light on the ability of biological systems to detect weak signals in a noisy environment.

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